An investigation of Entropy Generation inside the Boundary Layer over an Isothermal Wedge

Amir Malvandi¹, Faraz Hedayati², Davood Domiri Ganji³

¹Mechanical Engineering Department, Amirkabir University of Technology, amirmalvandi@aut.ac.ir
²Mechanical Engineering Department, Sari branch of Islamic Azad University, hedayati.faraz@gmail.com
³Mechanical Engineering Department, Sari branch of Islamic Azad University, dddg_davood@yahoo.com

Abstract
The steady two-dimensional boundary layer flow over a wedge, which is called Falkner–Skan flow is studied numerically to analyze the entropy generation inside the boundary layer. Constant wall temperature with no-slip boundary condition for velocity has been considered. Applying the transformation of governing equations including continuity, momentum and energy via similarity variables, a dimensionless equation for entropy generation inside the boundary layer is obtained for the first time. Analyzing the Bejan number, it is found that as Falkner–Skan power law parameter increases, the share of generated entropy due to fluid friction in total entropy generation increases. Finally, the effects of different parameters on the entropy generation are discussed and the physical interpretations of the results are explained in details.

Keywords: Entropy generation, Falkner–Skan, Boundary layer, Similarity solution,

Introduction
Boundary layers are thin regions next to the wall in the flow where viscous forces are important. The so called wall can be in various geometrical shapes. Blasius [1] studied the simplest boundary layer over a flat plate. The boundary layer flow over a static or a moving wedge in a viscous fluid (regular fluid) has been considered by Riley and Weidman [2] and Yacob et al. [3], which is an extension of the flow over a static wedge considered by Falkner and Skan [4]. They employed a similarity transformation that reduces the partial differential boundary layer equations to a nonlinear third-order ordinary differential equation before solving it numerically. A large amount of literatures on this problem has been cited in the books by Schlichting and Gersten [5] and Leal [6].

Beside the boundary layers, Entropy plays an essential role in our understanding of many diverse phenomena in many fields. It also form the basis of most modern formulations of both equilibrium and nonequilibrium thermodynamics. Entropy generation can be viewed as a measure of disorder or disorganization generated during a process, in other words it represents the addition thermodynamic irreversibility in a process which can causes more energy and power losses in the system so it is necessary to monitor this concept in each process to have a stable system. Bejan presented a method named Entropy Generation Minimization [EGM] in 1996 [7] to solve this problem. Other scientists have done some researches in this field too. Sahina et al. [8] investigated entropy generation in straight pipes and recently Tandiroglu [9] studied entropy generation for turbulent flow in a circular tube with baffle.

In this paper, for the first time, we have presented an equation for entropy generation inside the boundary layer for a wedge. Momentum, energy and entropy generation equations over a wedge are solved with Shooting method as a powerful numerical technique. Moreover, Diagrams are plotted and the physical interpretations of the results are discussed in details.

Mathematical Formulation
Consider an incompressible viscous fluid which flows over a wedge as shown in Figure 1. The wall temperature, $T_w$, is uniform and constant and is greater than the free stream temperature, $T_\infty$. It is assumed that the free stream velocity is $U_\infty(x) = U_\infty x^m$ where $U_\infty$ is uniform and constant as well and $0 < m < 1$ is the Falkner–Skan power-law parameter [4]. Further, assuming that the flow in the laminar boundary layer is two-dimensional and that the temperature gradients resulting from viscous dissipation are small, the continuity, momentum and energy equations in Cartesian coordinates can be expressed as:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]  

(1)
\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = U \frac{dU}{dx} + \nu \frac{\partial^2 u}{\partial y^2} \tag{2}
\]
\[
\frac{\partial T}{\partial x} + \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \tag{3}
\]

And the boundary conditions are:

At \( y = 0 \): \( u = v = 0 \) and \( T = T_w \)

\( \iota \text{m}_{y=w}, u = U_e(x) \) and \( \iota \text{m}_{y=w}, T = T_w \) \tag{4}

We look for a similarity solution of the Eqs.(1) and (2) with the boundary conditions Eq.(4) of the following form:

\[
\psi = \sqrt{\frac{2uxU_e}{m+1}} f(\eta) \\
\eta = \sqrt{\frac{(m+1)U_e}{2ux}} y
\]

Where \( x \) is measured from the tip of the wedge. \( \psi \) is the usual stream function, i.e. \( u = \frac{\partial \psi}{\partial y} \) and \( u = -\frac{\partial \psi}{\partial x} \)

and \( \nu \) is the kinematic viscosity of the fluid. Substituting Eqs. (5) and (6) into Eq.(2) we obtained the following ordinary differential equation

\[
f''''(\eta) + f(\eta)f''(\eta) + \left( \frac{2m}{m+1} \right)(1 - f'^2(\eta)) = 0 \tag{6}
\]

With these boundary conditions:

\[
f(0) = \frac{df(0)}{d\eta} = 0, \iota \text{m}_{y=\infty} \frac{df(\eta)}{d\eta} = 1
\]

The skin friction coefficient \( C_f \) can be defined as:

\[
C_f = \frac{\tau_w}{\rho U_e^2}
\]

Where \( \tau_w \) is the surface shear stress which is given by:

\[
\tau_w = \mu \frac{\partial u}{\partial y} \bigg|_{y=0}
\]

Substituting Eqs.(5),(6) into Eqs. (9) and (10) we obtain:

\[
\sqrt{\frac{2 \text{Re}_{in}}{m+1}} C_f = f''(0)
\]

Looking for Similarity solution for energy equation, Eq.3, we obtained:

\[
\theta'(\eta) + Pr f(\eta) \theta'(\eta) = 0
\]

Where

\[
\theta = \frac{T - T_w}{T_{w} - T_{\infty}}
\]

\( \theta(0) = 1, \iota \text{im}_{y=\infty}, \theta(\eta) = 0 \) \tag{13}

And also:

\[
Nu_x = \frac{xq_w}{k(T_w - T_{\infty})}
\]

Where \( q_w \) is the surface heat flux which is:

\[
q_w = -k \frac{\partial T}{\partial y} \bigg|_{y=0}
\]

Using Eq.(5), (6),(15) and (16) we obtain:

\[
\frac{(m+1) \text{Re}_{in}}{2} \theta(0) = -\theta'(0)
\]

**Entropy Generation**

Refer to Bejan [7], the volume rate of entropy generation can be expressed as:

\[
S''''_{gen} = \frac{k}{T^2} (\nabla T)^2 + \left( \frac{\mu}{T} \phi \right)
\]

Substituting Eqs.(5) ,6) and (13) into Eq.(21) we obtained a relation for entropy generation for a flow over a wedge:

\[
S = \frac{\theta'(\eta)}{2 \text{Pr Re} \varepsilon c(\theta_{w} + \theta(\eta))^2} \left( 2m+1 + \frac{\eta^2 (m-1)^2}{2 \text{Re}} \right)
\]

Where the dimensionless entropy generation may be defined by the following relationship:

\[
S = \frac{\nu^2 \Delta T}{U_e^4 \mu} S''''_{gen}
\]

In Eq.22, the first term is because of heat transfer and the second one is due to fluid friction. We call them \( S_h \) and \( S_f \) respectively and Bejan number is defined as follows:

\[
Be = \frac{S_h}{S_h + S_f}
\]

As it is obvious \( Be \) yields the share of \( S_h \) and \( S_f \) in total generated entropy, \( Be = 1 \) is the limit at which the heat transfer irreversibility dominates, \( Be = 0 \) is the opposite limit at which the irreversibility is dominated by fluid friction effects and \( Be = 0.5 \) is the case that the heat transfer and fluid friction entropy generation rates are equal.

**Result and Discussion**

The system of equations 6 and 11 with boundary conditions 7 and 12 has been solved numerically with Shooting method based on 4th order Runge-Kutta. In
addition, We have assumed that in Eq.18, $\theta_{\infty} = 2$, $Re = 1000$, $Pr = 1$, $Ec = 0.01$.

The variation of dimensionless velocity distribution for different values of $m$ is shown in Figure 2. When $m$ increases the pressure gradient along the streamline increases; therefore, $U_{\infty}$ and the boundary layer thickness decrease. The latter term will cause an increase in velocity gradients which increases the drag force on the wedge.

![Figure 2: Dimensionless velocity distribution for different values of $m$](image1)

The effects of $m$ on the velocity gradients inside the boundary layer have been shown in Figure 3. As mentioned before increasing in Falkner-Skan power law parameter causes increasing in velocity gradient on the surface, as a result, drag force on the wedge is increasing.

![Figure 3: Dimensionless velocity gradients for different values of $m$](image2)

Figure 4, shows The variation of dimensionless temperature distribution for different values of $m$. As thermal boundary conditions are constant, $m$ does not affect the temperature profile noticeably and just the momentum near the surface increases. This increase affects the heat transfer rate and the thermal boundary layer thickness decreases so the thermal gradient inside the boundary layer increases which has straight relation with heat transfer rate. This is shown in Figure 5.

![Figure 4: Dimensionless temperature distribution for different values of $m$](image3)

Figure 5 shows that increasing in $m$ increases the generated entropy along the boundary layer. We can divide each curve into two parts. First, a gentle increase in generated entropy which is a result of gentle increase in velocity and temperature gradients at the beginning (Figures 3 and 5) and second, noticeably decrease in generated entropy which can be explained by a marked decrease in velocity and temperature gradients.

![Figure 5: Dimensionless temperature gradients for different values of $m$](image4)

Figure 6 shows that increasing in $m$ increases the generated entropy along the boundary layer. We can divide each curve into two parts. First, a gentle increase in generated entropy which is a result of gentle increase in velocity and temperature gradients at the beginning (Figures 3 and 5) and second, noticeably decrease in generated entropy which can be explained by a marked decrease in velocity and temperature gradients.

![Figure 6: Dimensionless temperature distribution for different values of $m$](image5)
Variation of total entropy generation with $m$ is shown in Figure 8. As discussed before, increasing in $m$ causes increasing in thermal and velocity gradients so we can see a noticeable increase in total entropy generation inside the boundary layer.

![Figure 6: Volumetric entropy generation distribution for different values of m](image)

![Figure 7: Bejan number for different values of m](image)

**Conclusion**

In this paper, we have studied entropy generation inside a steady two dimensional boundary-layer flow over an isothermal wedge numerically. In addition the generated entropy over a wedge is formulated and studied inside the boundary layer for the first time. All equations are solved numerically with using Shooting method as a powerful technique. The effects of different wedge slopes, on the velocity and temperature profiles and the generated entropy inside them are discussed. Be Number is plotted and its variation is discussed as well. It is found that when $m$ increases entropy generation in the boundary layer increases. Moreover the share of fluid friction in total entropy generation climes up but in overall, generated entropy due to heat transfer dominates.

![Figure 8: Total entropy generation for different values of m](image)

**References**


